Theoretical Treatment of Drift Waves in A Collisionless Plasma Regime within A Magnetically Confined Plasma

معالجة نظرية للموجات المنحرفة في نظام البلازما الغير تصادمية المنحصرة في مجالات مغناطيسية

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Theoretical Treatment of Drift Waves in A Collisionless Plasma Regime within A Magnetically Confined Plasma

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Abstract

The main objective of this study is to show that, the drift wave frequency and the radial distribution of the oscillating electric potential of a cylindrical magnetized plasma are influenced by the gradients in electron temperature, and the $\vec{E} \times \vec{B}$ rotation.

A theoretical model is introduced in this study which is based on two-fluid equations, for either electrons or ions and simplified to the collisionless magnetised plasma regime. In the present work, that the inclusion of electron temperature gradient is essential and must be taken into account in the theory. In addition, the $\vec{E} \times \vec{B}$ plasma rotation which arises when a radial electric field exist in an axial magnetic field is included into a theoretical model. The effect of rotation will Doppler shift the drift wave frequency so that plasma rotates as a solid body. The radial plasma density and temperature profiles which are required as input to the theoretical calculation are also presented and displayed. The theoretical model predicts the actual frequency, and the radial fluctuation profiles of the mode and hence, the position of the maximum wave amplitude can be determined. We made a comparison of experimental measurement with the theoretical prediction for radial variation of eigen function of density fluctuation are also presented. The experimental data is obtained from an axial magnetised plasma of Zhang L. (1992) results. A matlab computer calculation is used for the differential equation for the radial distribution of the eigen function of density fluctuation. The results illustrates the importance of the effect of the electron-temperature and $\vec{E} \times \vec{B}$ rotation Doppler shift into a theoretical model.

**Keywords:** The Doppler Shift, The $\vec{E} \times \vec{B}$ Rotation, Drift Waves.
الملخص

تهدف الدراسة إلى توضيح أن تردد الموجات المنحرفة لشعاع البلازما عبر الحد الكهربائي في نموذج البلازما الإسطواني الممغنط يتأثر بالتدرج الكثافة الإلكترونية وحد الدوران $\vec{E} \times \vec{B}$، حيث يستند النموذج النظري على معادلات الحركة two-fluid للأيونات والإلكترونات، وتشتمل هذه الدراسة على تركيز التغيير في درجة الحرارة وكذلك دوران $\vec{E} \times \vec{B}$ للبلزما التي تنشأ عندما يوجد تغير في المجال الكهربائي، وتم الحصول على النتائج النموذجية في النموذج النظري، فإن دوران دوبلر الناتج من تأثير $\vec{E} \times \vec{B}$ قد يساهم في انحراف التردد.

وقد عرضت دراسة تأثير التغير في درجة الحرارة وكذلك التغير في تردد الموجة واتساعها كدالة في مسافة شعاع البلازما. وتم القيام بمقابلة بين النظري والتحقيق حيث تم الوصول إلى تأثير التغير الناتج من تأثير $\vec{E} \times \vec{B}$ في النموذج النظري.

كلمات مفتاحية: البلازما – الطاقة – الموجات المحصورة.
Dedication

To my dear father, mother, brothers and sisters.
To my homeland Palestine the only place in which I feel alive.

Soboh Al Qeeq
Acknowledgment

I am extremely indebted to my supervisor Dr. Samir S. Yassin for his constant support and help, stimulating suggestions and encouragement which helped me during all time of this research as well as, arranging and writing of this thesis. I would like also to express my thanks to the members of staff in the Department of Physics, Islamic University of Gaza. My sincere gratitude also extends to those who helped me to pursue this work.

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Researcher:
Soboh Al Qeeq.
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Chapter 1
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1.1 Background

Drift waves and drift instabilities occupy a special place in the spectrum of collective plasma processes. This is because they can occur in all plasma which are not in equilibrium as is the case of confined plasma. Thus under laboratory conditions gradients in temperature, density, magnetic field, and instability are inevitable. Wherever the gradient exists, a plasma current or particle drift exists, drift waves are supported by these gradients, and instabilities can tap the energy in the drift waves.

Many theoretical and experimental observation show the drift waves appear in a plasma in several shapes, depending upon the plasma parameters. Such waves are electrostatic oscillation which propagate mainly perpendicular to both magnetic field and the gradients. This perpendicular propagation is in the direction of the electron diamagnetic drift and the phase velocity of the wave is of the order of drift velocity. The overall appearance of the drift mode and its harmful effect on plasma confinement have made it a vital importance for both theoretical and experimental study. Briefly, drift waves lead to plasma losses from the confinement system which we can not avoid, that is why an extensive study has been made of drift waves in many devices.

A spark in a gas will create a plasma. A hot gas passing through a big spark will turn the gas stream into a plasma that can be useful. Plasma torches like that are used in industry to cut metals. The biggest chunk of plasma you will see is that dear friend to all of us, the sun. The sun's enormous heat rips electrons off the hydrogen and reaction helium molecules that make up the sun. Essentially, the sun, like most stars, is a great big ball of plasma, Rosenbluth (1968).

Historically, the drift waves have provided considerable impetus to the attempt to analyze plasma stability through the Vlasov equation rather than through various moment equations. This is because resonance phenomena are so important to the
instability process and many of the problems demand consideration of wavelengths smaller than the ion Larmor radius, Horton, Varma (1972).

Any thing that causes drift affects the problem, and the drift due to curvature of the confining magnetic field has a profound effect on these motions. In some cases the curvature is the source of the instability, Horton. et al. (1981) but in most cases curvature is included in order to estimate the stabilizing effect of a magnetic well.

1.2 Previous Studies

Any ionized gas cannot be called a plasma; there is always some degree of ionization in any gas. A useful definition of a plasma is:

A plasma is a quasi-neutral gas of charged and neutral particles which exhibit collective behavior. Quasi-neutral means that the ion density and electron density are equal to each other. Collective behavior means that, the charged particles can generate local concentrations of positive or negative charge, which give rise to electric fields.

Motion of charges also generates currents, and hence magnetic fields. These fields affect the motion of other particles away, Rosenbluth (1962). One of the earliest accounts of the instability caused by ion temperature gradients is found in the paper by Rudakov and Sagdeev (1961), where it is shown that the growth of the "ionic electrostatic "wave is caused by" a continuous inflow of heat from a region with high unperturbed temperature into the region where the temperature is rising on account of the compression due to the plasma wave under the conditions of zero density gradient \( n(x) = \text{const.} \) and finite temperature gradient \( T_i(x) = T_e(x) \neq \text{const.} \).

More detailed discussions on the fluid and kinetic models of this instability, the dispersion relation, the critical value of \( \eta_i \) for the marginal stability and localization of the mode are presented by Kadomtsev and Pogutse (1969) based on the local approximation. Coppi, Rosenbluth, and Sagdeev (1976) have derived the integral equation of the instability due to ion temperature gradients from the Vlasov equation, using the normal mode analysis and the fluid limit of this kinetic model is also discussed.
On the other hand, the model equations for instabilities of the collision drift wave due to ion and electron temperature gradients are derived and discussed by Hinton and Horton (1971) and also by Horton and Varma (1972), where it is based on the two-fluid equations, Braginskii (1965) with the effects of resistivity, viscosity and thermal conductivity. The ion temperature gradient driven drift instability, modeled in a slightly different way from previous model equations and called the ion-mixing mode by Coppi and Spight (1978). It is used to explain the rate of density rise observed when neutral gas is fed into a tokamak plasma during a stable discharge, Coppi and Spight (1978), Antonsen et al. (1979).

Simple fluid model equations of drift waves in the absence of the ion and electron temperature gradients are derived in the collisionless limit by A. Hasegawa and Mima, (1978) and in the collision limit by Hasegawa and Wakatani, (1983), Wakatani and Hasegawa (1990). These sets of equations provide simple models of plasma turbulence, from which one can relatively, easily perform mode-coupling analyses and study plasma-turbulence properties such as wave number spectra. Horton, (1992), has also presented a simple set of fluid equations of the ion temperature gradient driven turbulence in order to assess the anomalous ion thermal transport. In their model, the electron-temperature-gradient effects are excluded and only the three scalar fields of fluctuations, the electric potential $\varphi$, the ion pressure $P$ and the parallel ion velocity $v_{\parallel}$, are involved. Several other simple fluid models have been proposed for the study of drift wave turbulence under various conditions, Terry and Horton (1983).

There are two different branches of the ion temperature gradient driven mode. One is called "slab type", which is the drift wave coupled with the ion acoustic wave that is destabilized by the local ion temperature gradient. The other is called "interchange type, Hamaguchi and Horton (1990), Horton (1981), which is destabilized by bad curvature of the magnetic field lines in the presence of the finite ion temperature gradients, Terry and Horton (1983).

Short wavelength ion temperature gradient instability in toroidal plasmas, series of ion temperature gradient (ITG or $\eta_i$) driven modes in the short wavelength
region, $|k_\perp \rho| > 1$, are also investigated with a gyro kinetic integral equation code in toroidal plasmas. These instabilities exist even if electrons are assumed adiabatic.

Four wave-nonlinear coupling of a large amplitude whistler with low frequency drift wave and whistler wave sidebands is also examined. The pump and whistler sidebands exert a low frequency ponder motive force on electrons introducing a frequency shift in the drift wave. The drift waves of low transverse wavelength tend to be destabilized by the nonlinear coupling. Oblique propagating whistler pump with transverse wave vector parallel to $k_\perp$ is also effective but with reduced effectiveness, DuBois (2012), Pawan Kumar and Tripathi (2013).

The study is also considered the ionization and recombination of the ions, as well as the collision between ions and dust grains. The damping solitary wave solution in a dusty plasma have been previously studied and found that the damping rate of the solitary wave increases as both the mass and the density of the dust grains increase. However, it decreases as the charge of the dust grain increases Xiao (2006), Xue Yang (2013).

The plasma is sometimes referred to as the “fourth” state of matter. When a solid is heated sufficiently that the thermal motion of the atoms break the crystal lattice structure apart, usually a liquid is formed. When a liquid is heated enough that atoms vaporize off the surface faster than they recondense, a gas is formed. When a gas is heated enough that the atoms collide with each other and knock their electrons off in the process, a plasma is formed Zhe Gao et al. (2005), Horton and Varma (1972).

### 1.3 Basic description of drift waves

Waves are electrostatic oscillations which propagate mainly perpendicular to both the magnetic field and the gradients. This perpendicular propagation is in the direction of the electron diamagnetic drift and the phase velocity of the wave is of the order of drift velocity. In addition, drift modes may also propagate in the axial direction parallel to the magnetic field and possess a longitudinal wavelength $\lambda_\parallel$. However, since any confined plasma must have density gradients, it was first thought
that drift waves were unavoidable and hence, these waves have been properly termed "universal" instabilities.

1.3.1 Physical model

It is worthwhile to review the mechanism of excitation of drift waves in an inhomogeneous plasma. Consider a constant and uniform magnetic field containing plasma with density, and pressure gradients perpendicular to the magnetic field. The cold ions are distributed so that a constant density gradient exists in the negative x-direction, and isothermal electrons are distributed so that the electric field is zero.

The important parameter of physical processes are the electric field drifts of both species in the direction of the density gradient, and the motion of the electrons along the field lines. Fig. (1.1) shows the evolution of a drift wave propagating across a density gradient $\nabla n_0$. If a density perturbation develops around a field line, the region of positive density perturbation has been shaded, at time the potential $\varphi$ will be distributed as in fig. (1.1a). Since the electrons flow freely along B, they must be in equilibrium along line of force and obey the Boltzmann relation:

$$\frac{n}{n_0} = \frac{e\varphi}{k_B T_e}$$

(1-1)

Provided only that $k_\parallel \neq 0$ and $\frac{\omega}{k_\parallel} \leq v_{th e}$ the electron thermal speed. The ion density perturbation must also have the same form at $t = 0$, where the condition of charge neutrality requires that the electron and ion densities must be equal.

The distribution of potential \( \varphi \) gives rise to an electric field \( E_y \) in the direction of the arrows, parallel and anti-parallel to its wave vector. Thus, a drift \( v_e = \frac{E_y}{B} \) of both electrons and ions will be established in the \( x \)-direction and opposite to the direction of the density gradient. This drift convects plasma of different density and temperature into a field line with the phase of the wave. A little later in time, in the regions where the electric field drifts in the opposite direction to the density gradient, particles drifting from a denser part of the initial distribution, so an accumulation of density starts to build up. Thus, the perturbation in potential and density will move upwards on the diagram by a quarter cycle in the same direction as the diamagnetic drift velocity. This is shown in fig. (1.1b). A drift wave moves back and forth in the \( x \)-direction as well as in the \( y \)-direction.

To find the magnitude of the phase velocity, we note that the rate at which \( v_e \) increases the density at any given \( x \) is:

\[
\frac{\partial n}{\partial t} = -v_e \frac{dn}{dx} = -i \omega n
\]  

(1-2)

This is just the equation of continuity, and since:

\[
E_y = -ik \perp \varphi \quad \text{and} \quad \varphi = \frac{k_b T_e}{e n} \frac{n}{n_0}
\]  

(1-3)

We have:

\[
v_e = \frac{E_y}{B} = -ik \perp \frac{k_b T_e}{e B} \frac{n}{n_0}
\]  

(1-4)

Hence:

\[
-i \omega n = ik \perp \frac{k_b T_e}{e B} \frac{n}{n_0} \frac{dn}{dx}
\]  

(1-5)

Then the phase velocity of the wave propagating in the \( y \)-direction is:

\[
\frac{\omega}{k \perp} = -\frac{k_b T_e}{e B} \frac{n}{n_0} = v_{De}
\]  

(1-6)

Where \( T_e \) is the electron temperature, \( B \) is the axial magnetic field, \( n_0 \) is the equilibrium plasma density, \( k_b \) is Boltzmann's constant and \( e \) is the electronic charge.

The energy source for the instability is the free energy of confinement the plasma. As the presence of a pressure gradient is unavoidable in a confined plasma, the drift instability is often referred to as the "universal" instability. However, more recent evaluation shows that shear in the magnetic field can stabilize the mode.
1.4 General review

As many theoretical investigations and experimental observations show the drift instability appears in a plasma in several guises; depending upon the plasma parameters such as collision frequency, $\beta$ (ratio of plasma pressure to magnetic pressure), ratio of ion to electron mass, degree of ionization, and the magnitude of the density gradient scale length relative to the ion lamor radius, and the length of the device.

In cylindrical devices with axial magnetic fields, extensive experiments in collisional, or collisionless plasma have resulted in the observation of oscillation localized in or near the region of maximum density gradient, and propagating with an azimuthal velocity comparable to the electron diamagnetic drift velocity, Handel et al (1968); Ellis et al (1980). These waves were identified as density-gradient-driven drift waves.

The wave maximum is a large fluctuation of the equilibrium density, and potential. These large perturbations originate in small, random density and potential fluctuations occurring in the equilibrium state of the plasma; Marden – Marshall et al. (1986). The modes have an integral number, $m$, of maxima propagating azimuthally, with $m = 1, 2, 3$ and 4 most commonly observed in cylindrical plasma column. Propagation of the modes is primarily azimuthal, and is in the direction of the electron diamagnetic drift:

$$v_{De} = -\frac{k_B T_e}{eB} \frac{1}{n_0} \frac{dn_0}{dr} \hat{\theta}$$

(1-7)

where $\hat{\theta}$ represents a unit vector $\perp \hat{B}$.

Most early theoretical interest for drift waves occurring in cylindrical plasma made use of the local slab model, Hendel et al (1967, 1968); Van Andel et al (1977). This model uses the fluid equation in a slab geometry and assumes that radial localization of the mode is confined to a region which is much smaller than the density gradient scale length. In addition, the oscillation is assumed to have a sinusoidal dependence. Thus, the perturbed density would have the form:

$$\tilde{n} = n(r) e^{i(k_xx + k_yy + k_zz - \omega t)}$$

(1-8)

Thereby simplifying the calculations considerably.
While the assumption of axial and azimuthal normal modes is experimentally justified in a cylindrical geometry, the assumption of a radial normal mode is not. This led Chen (1967) to a cylindrical treatment in which the arbitrary radial dependencies were allowed, i.e.

\[ \tilde{n} = n(r)e^{i(m\theta + k z - \omega t)} \]  

(1-9)

In addition, the local approximation was not made. This model could predict both the frequency and growth rate of the instability as well as its radial shape. The latter demonstrated that the wave maximizes where the density gradient is largest, as expected.

Both the slab and cylindrical models, even in their least rigorous treatments, demonstrate that the frequency of oscillation is very nearly equal to the drift frequency \( \omega^* \), where

\[ \omega^* = \frac{m}{r} v_D e - \frac{m k_b T_e}{e B} \left( \frac{1}{r n_0} \frac{dn_0}{dr} \right) \hat{\theta} \]  

(1-10)

It is clear that the radial density profile is an important input into the theory, and a suitable choice can simplify it considerably. Thus, if a Gaussian density profile is assumed, then the quantity in brackets is constant, and if the plasma is also isothermal, then \( \omega^* \) is constant. This is, in fact, the usual approach of Chen (1984) and Chu et al (1969). Since density profiles are not necessarily best approximated by Gaussians, a cylindrical model was developed by Ellis et al. (1980) that allowed for arbitrary density profiles.

Usually, drift waves are observed in a rotating plasma column. Marden, Marshall et al. (1986) have shown that the plasma rotation arises when a radial electric field, \( E_r \), coexists with an axial magnetic field, \( B_z \), and \( f_{rot} \) is at the frequency:

\[ f_{rot} = \frac{1}{2 \pi r} \frac{E}{B} \]  

(1-11)

This rotation is non-uniform, or sheared, whenever \( B \) is constant and \( E \) does not vary linearly with the radius.

In spite of the fact that the drift wave maximizes in a region of large density gradient, it have identified, in their view, another class of low frequency oscillations as drift waves. These waves were found largest at the (radial) edges of Q-machines, where the density is negligible; but potential gradients are large. These "edge" oscillations were later identified as Kelvin-Helmholtz modes. Since a radial potential gradient, in
combination with an axial magnetic field, gives rise to an $\vec{E} \times \vec{B}$ rotation of the plasma column, then the kelvin- Helmholtz instability establishes when the electric field varies radially, causing a non-uniform rotation.

A cylindrical two-fluid model has been developed for the Kelvin- Helmholtz instability, in which the effect of rotational shear and centrifugal force could be isolated. In rotational shear and centrifugal force could be destabilizing, or capable of enhancing the mode evolution. Their results have demonstrated the effect of a density gradient on a low frequency instability which develops from a sheared potential gradient.

In most instances, in the drift wave literature, Ellis et al. (1980), the rotation is treated as uniform, so that the plasma rotates as a solid body. The effect of a sheared potential gradient on a low frequency instability that develops from a density gradient, in other words the effect of nonuniform column rotation on a drift wave, was investigated by Marden – Marshall et al. (1986). They showed that the radial electric field is strongly suggested as the destabilizing mechanism for the drift instability. The rotating column could be thought of as a moving source from which azimuthally travelling waves are emitted. However, it was realized that the drift wave frequency in the laboratory frame will differ from its value in the frame of the plasma. In other words, it is clear that the moving column will Doppler shift the drift wave frequency. The observed frequency will be either the sum of the drift wave and rotational frequencies (for rotation in the direction of propagation), or the difference of these frequencies (for rotation opposite to the direction of propagation), see fig. (1.2). Thus, the effect of plasma column rotation in any drift wave model is simplest when the rotation is uniform. It has a marked effect by destabilizing the drift waves.

![Figure 1.2: Cylindrical plasma column. Density and potential gradients give rise to the electron diamagnetic and drifts, $v_{De}$ and $v_E$ respectively, Hutchinson I. H., (2001).](image-url)
1.4 Objective of this study

The main objective of this study is to show that the drift wave frequency and the radial distribution of the oscillating electric potential of a cylindrical magnetized plasma are influenced by the gradients in electron temperature and the $\vec{E} \times \vec{B}$ rotation. To reach the objective:

1- The theory is formulated by two-fluid hydrodynamical equations, taking into account the electron-temperature oscillation, the radial variation of density and temperature, the $\vec{E} \times \vec{B}$ rotation, and the electron motion parallel to the magnetic field lines.

2- This theory in the form of a second order differential equation for the oscillating electric potential as an eigenfunction and the drift wave frequency as an eigenvalue where the dispersion relation that represent the considered plasma regime will be obtained.

3- Comparison between the theoretical model and an experimental data of helium plasma from recent published papers using matlab program integration method will be carried out throughout this work.

1.5 Expected Outcome

The development and analysis of the drift wave theory of collisionless plasma regime will be reported based on the two – fluid equation. Dispersion relation that describes the drift wave in a collisionless plasma regime, where in density temp., pressure gradients and $\vec{E} \times \vec{B}$ rotation will be investigated.

The theory is applied to an experimental data of hydrogen plasma, the data is obtained from an axial magnetised plasma of linear system, Zhang L. (1992) results plasma density and electron temperature profiles as well as the radial eigen function of potential are displayed through the present work. Using matlab integration method, our calculation will show the importance of the temperature variation and the $\vec{E} \times \vec{B}$ rotation in the predictions of drift wave frequency and radial position of the maximum wave amplitude.
Chapter 2

Literature Review
Chapter 2
Literature Review

The goal of the present work is to use the model of the two-fluid equation for describing the drift wave vortices in a sheared magnetized plasma with low beta and finite ion temperature gradient. The dispersion relation of the slab-type ion temperature gradient driven mode will be obtained and determined for both collisionless and collisional drift waves in sheared magnetized plasma.

The outline of this chapter is as follows: At first magnetic containment in section 2.1, while plasma conductivity in section 2.2, then ablation driving mechanisms in section 2.3 and the characteristics of drift wave in section 2.4, then presents the mechanism of drift wave in section 2.5, Finally physical mechanism of the rayleigh-taylor instability in section 2.6.

2.1 Magnetic containment

The fact that the orbit of a charged particle in a magnetic field, B, is a spiral about a field line enables magnetic fields to be used for isolating the hot plasma form the walls of the containment vessel. We can think of the plasma as a collection of charged particles, each with its individual orbit, or as an electrically conducting fluid, Hugill, (1981). Open magnetic traps where the field lines emerge from the containment region. Particles moving along the field are reflected by a region of higher field strength called amagnetic mirror. Such a device cannot contain a maxwellian distribution of particle velocities so that collisions between particles will lead to losses as their velocity vectors are scattered in the direction along B. The loss rate is very high except at very high temperature, but it seems feasible to recirculate some of the power lost by direct conversion of the escaping plasma flux to electricity, Hugill, (1981).

Closed line configurations in which the field lines close on themselves within the containment region are topologically toroidal and are at present the favoured candidates for an economic reactor system, in spite of the extra technological difficulties associated with toroidal linkage. The purely toroidal field component must be augmented by a field around the toroidal axis called the poloidal field which produces a set of nested toroidal magnetic surfaces, Hugill, (1981).
2.1.1 Magnetized plasma

Plasmas are complicated because motions of electrons and ions are determined by the electric and magnetic fields but also change the fields by the currents they carry.

Equation of motion:

\[ m \frac{dv}{dt} = e(\vec{E} + \vec{V} \times \vec{B}) \]  

(2-1)

You can solve the differential equation using many variables into the equation as needed so, to get position \( r \) and velocity \( (v = r') \) given \( E(r, t), B(r, t) \).

2.1.2 Uniform B field, \( E = 0 \)

\[ m \frac{dv}{dt} = e(\vec{V} \times \vec{B}) \]  

(2-2)

In the plane perpendicular to \( B \):

![Circular orbit in uniform magnetic field](image)

Figure (2.1); Circular orbit in uniform magnetic field, Hutchinson I. H., (2001).

Acceleration is perpendicular to \( v \) so particle moves in a circle whose radius \( r_L \) is such as to satisfy, Hutchinson I. H., (2001):

\[ m r_L \omega_c^2 = \frac{m v_{\perp}^2}{r_L} = \pm e \, v_{\perp} \, \vec{B} \]  

(2-3)

\( \omega_c \) is the angular frequency. It gives, Hutchinson I. H., (2001):

\[ m \omega_c^2 \left[ v_{\perp} \frac{1}{\omega_c} \right] = \pm e \, v_{\perp} \, \vec{B} \]  

(2-4)

Particle moves in a circular orbit with angular frequency \( \omega_c = \pm \frac{eB}{m} \) the "Cyclotron Frequency" and radius \( r_L = \frac{v_{\perp}}{\omega_c} \) the "Larmor Radius".
2.1.3 Uniform B and non-zero E

\[ m \frac{dv}{dt} = e(\vec{E} + \vec{V} \times \vec{B}) \]

Parallel motion: Before, when \( E = 0 \) this was \( v_\parallel = \text{const.} \).

Now it is clearly:

\[ v_\parallel = \frac{e E_\parallel}{m} \quad (2-5) \]

Constant acceleration along the field. Perpendicular motion:

![Diagram showing \( \vec{E} \times \vec{B} \) drift orbit](image)

Figure (2.2): \( \vec{E} \times \vec{B} \) drift orbit, Hutchinson I. H., (2001).

Speed of positive particle is greater at top than bottom so radius of curvature is greater. Result is that guiding center moves perpendicular to both \( E \) and \( B \). It 'drifts' across the field.

It is clear that if we can find a constant velocity \( v_d \) that satisfies:

\[ \vec{E} + \vec{v}_d \times \vec{B} = 0 \quad (2-6) \]

Take \( \times B \) the above equation, we get:

\[ \vec{E} \times \vec{B} + (v_d \times \vec{B}) \times \vec{B} = \vec{E} \times \vec{B} + (v_d \cdot B)B - B^2 v_d = 0 \quad (2-7) \]

So that:

\[ v_d = \frac{\vec{E} \times \vec{B}}{B^2} \quad (2-8) \]

Hence the full solution is:

\[ v = v_\parallel + v_d \quad (2-9) \]

\( v_d \) is the "\( \vec{E} \times \vec{B} \) rotation drift waves" of the gyrocenter. Comments on \( \vec{E} \times \vec{B} \) drift:

1. Hence it is in the same direction for electrons and ions.
2. It is independent of the properties of the drifting particle \((e, m, v)\).

3. Formula given above is exact except for the fact that relativistic effects have been ignored. They would be important if \(v_d \sim c\).

4. Underlying physics for this is that in the frame moving at the \(\vec{E} \times \vec{B}\) drift \(E = 0\). We have ‘transformed away’ the electric field.

### 2.2 Plasma conductivity

If a static electric field is applied to a plasma, the ions being more massive remain almost stationary while the electrons are accelerated, but lose momentum by colliding with ions, or with neutrals, so that a steady drift velocity \(V\) is established, Hastie, (1981):

\[
nmV\nu = neE
\]  
(2-10)

Where \(\nu\) is a collision frequency for momentum interchange and so with the momentum transfer cross-section, the number density of scattering centres, and the electron thermal velocity (assumed much greater than \(V\)) Hastie, (1981).

Now, the current density is given by:

\[
J = neV = \frac{ne^2}{mv} E
\]  
(2-11)

So that the conductivity is given by

\[
\sigma = \frac{ne^2}{mv}
\]  
(2-12)

#### 2.2.1 Weakly ionized plasma

In a weakly ionized plasma the electron collisions are predominantly with neutrals, \(n_s = n_0\) the neutral density, and:

\[
\sigma = \frac{ne^2}{m\sigma e n_0 \nu_e}
\]  
(2-13)

The conductivity is proportional to the electron density \(n\), Hastie, (1981).

#### 2.2.2 Fully ionized plasma

In a fully ionized plasma the electron collisions are with ions, so \(n_s = n_i = n\) (for singly charged ions), and, Hastie, (1981):

\[
\sigma = \frac{e^2}{m\sigma e \nu_e} = \frac{\left(kT\vec{E}(4\pi\varepsilon_0)^2}{e^2m^2}
\]  
(2-14)
(On using the Rutherford formula \( \sigma_e \propto e^4/(4\pi\varepsilon_0v_e^2m)^2 \). Thus the conductivity is independent of the electron density \( n \), and is proportional to \( T^{3/2} \). This fact means that ohmic heating of a plasma in a fusion device by inductively drawing a current is at its most efficient at low temperatures, and becomes ineffective as fusion temperatures \( \geq 10 \text{ keV} \) are approached, Hastie, (1981).

2.3 Ablation driving mechanisms

In principle, we may use any device, which is capable of delivering energy to the surface of the target element at a suitably high rate to drive the ablation. In practice, the high peak powers and short pulses required have restricted consideration to three possibilities, Pert G.J, (1981).

2.3.1 Electron beams

The use of large energy electron beams incident directly on the target provides a source of considerably greater total energy than present day lasers. Such devices do not, however, readily allow temporal pulse shaping or tight focusing, and furthermore require careful design to avoid pre-heat of the core. Their application is therefore limited to relatively massive, and probably inefficient targets. Pert G.J, (1981).

2.3.2 Ion beams

The most recent suggestion involves the use of ion beams as a driver. Two approaches have been considered. The acceleration of light ions (protons) to MeV energies with current densities of order 1MA/cm can now be achieved using conventional high voltage pulsed line techniques developed for electron beam research. The upgrading of these sources by a factor of 10 to 100 necessary for fusion breakeven is considered feasible, and current work at sandia laboratories in the united states is directed to this end. Major problems involving beam focusing and particle interaction physics, however, remain to be solved in a reactor context, Pert G.J, (1981).

Alternatively one may consider an approach using heavy ions (uranium) of high energy (GeV), but relatively low current, generated in a conventional high energy accelerator of high efficiency. The interaction physics and beam generation and focusing of this latter approach are expected to be well understood, but the very large initial capital cost of the experiments has limited study to theoretical analysis at present. Pert G.J, (1981).
2.4 Characteristics of the drift wave

Drift waves can be observed experimentally as a fluctuation in the plasma density, or plasma potential with \( \frac{e\phi}{k_B T_e} \) representing the corresponding normalized perturbation. These quantities of perturbation differ in phase, Hendel and Politzer (1968). Both quantities fluctuate in time and in space. Therefore, waves in plasma are characterized by their amplitude, frequency \( (\omega) \) and wave number \( (k) \). Politzer said that "The modes can be identified as drift waves by means of measurements of the dependence of \( \omega \) and \( k \) on the externally variable parameters; density, temperature, ion mass and magnetic field ". Of these the magnetic field, pressure and density give the widest range of variation.

The propagation direction of the drift wave in the plasma frame is primarily azimuthal \( (\theta) \) in the same direction of the electron diamagnetic drift velocity(\( i.e. \ as \ \vec{E} \times \vec{B} \)). This is also the direction of the plasma rotation due to the radial electric field which is directed towards the walls. Strictly the observed modes will therefore be at a Doppler shifted drift wave frequency, since the drift wave is propagating in a moving plasma. However, as we have seen our present estimates of the Doppler shift are not reliable enough to be used. The direction of propagation and the mode frequency are determined by a radial density profile which is decreasing outward.

One of the criteria for excitation of drift waves is the voltage applied to the grid, with which we control the state of the plasma. Ellis et al. (1980) reported that the application of a D.C voltage bias to the grid can either serve to suppress or excite the drift waves; depending on the exact magnitude and polarity of the bias, with respect to the machine structure as ground. In the present work, drift waves have been experimentally studied in terms of grid current. The waves are generally found to be excited with grid current.

Several modes are obtained theoretically under our conditions, relevant to the maintenance of a collisionless plasma regime. These are likely to be modes with different azimuthal mode numbers. The frequency spectrum of the ion saturation current fluctuations, which are representative of the perturbations in the plasma density
are determined. Also, This would suggest that the density gradient is the important factor to excite drift waves.

Most of the observed modes are coherent, that is all the wave power centered on, or very near, a single frequency in range of 60-90 KHz. The modes have large fluctuation levels ($\bar{n}/n_0 = 26\% \text{ peak to peak amplitude}$) which maximize near mid-radius of plasma beam, where the density gradient is largest. This ratio is the fluctuating component of the total ion saturation current $\bar{I}$ (or $\bar{n}$), normalized to the total DC ion-saturation current $I$ (or $n$). Motley (1975) indicated that the radial wave function has amplitude which is zero at $r = 0$, peaks with increasing distance $r$ where the density gradient is large; and falls to zero again at the edge of plasma beam.

Drift waves are also observed in the axial direction. Two single probes were positioned at the same radius and azimuth, but at different axial location, display signals which are coherent, so that the phase relationship may be established. One probe across the plasma beam is used as a reference, and the other moves axially along the system.

2.5 The Mechanism of Drift Wave
2.5.1 Plasma convection in quasineutral disturbances

Consider the $\vec{E} \times \vec{B}$ convection of plasma around a local excess of positive ion density $\delta n_i$ as shown in Fig. (2.3). The direction of the background density gradient is $x$ and the depth of the disturbance is $\delta x$ the symmetry direction is $y$, and the length of the disturbance is $\delta y$.

The confining magnetic field is in the $z$ direction. For slow dynamics, the electrons neutralize the excess ion charge by flowing along the magnetic field to establish the local Boltzmann density distribution:

$$n_e = N \exp(e\varphi/k_B T_e)$$  \hspace{1cm} (2-15)

Where $\varphi$ is the electrostatic potential created by the charge imbalance and $N(x)$ the ambient density in the absence of the perturbation ($\varphi = 0$). Here $k_B$ is the Boltzmann constant and $T_e$ the electron temperature.
In general the perturbed potential is determined by Poisson’s equation:

\[ \nabla^2 \phi = -4\pi e \left( \delta n_i - \delta n_e \right) \]  

(2-16)

In this self-consistent field problem \( \delta n_a = \delta n_a(\vec{x}, \varphi(x,t)) \). For structures that are large compared to the electron Debye length \( \lambda_D = \left( \frac{\varepsilon_0 k_B T_e}{n_0 e^2} \right)^{\frac{1}{2}} \); however, the fractional charge separation allowed by the Poisson equation is small. From Eq. (2-16) the fractional deviation of electron and ion density are:

\[ \frac{\delta n_i - \delta n_e}{N} = \left( \frac{-k_B T_e}{4\pi Ne^2} \right) \nabla^2 \left( \frac{e \varphi}{k_B T_e} \right) \]  

(2-17)

\[ \leq \left( \frac{\lambda^2_{De}}{\delta x^2} \right) \left( \frac{e \varphi}{k_B T_e} \right) \ll 1 \]  

(2-18)

Thus the principle of quasi-neutrality is applied. In the quasineutral regime, the plasma potential adjusts its value to make \( n_i(\varphi) = n_e(\varphi) \) to the first order in \( \lambda^2_{De}/\delta x^2 \). The resulting small fractional charge separation computed from Poisson’s equation using the potential rather than vice versa. All the drift-wave dynamics discussed in this review are in the regime of quasi-neutrality where

\[ \rho_0 = \sum a q_a n_a = 0 \]  

(2-19)

Determines the evolution of the electrostatic potential and the plasma Eigen modes. Here \( a \) is the species index for particles of charge \( q_a \) and density \( n_a \). We
return to analyze the convection in Figs. (2.3) and (2.4), Horton. (1997), Shenggang. (2002), after discussing the general features of the fluctuations.

The condition for the electrons to establish the Boltzmann distribution in the drift wave ion acoustic wave disturbances follows from the parallel electron force balance equation. For fluctuations with the parallel variation $k_\parallel = 2\pi/\lambda$ sufficiently strong so that electrons with thermal velocity $v_e = (T_e/m_e)^{1/2}$ move rapidly along $k_\parallel$ compared with time rate of change of the fields $\omega < k_\parallel v_e$ the dominant terms in the electron fluid force balance equation

$$m_e n_e \frac{dv_\parallel}{dt} = -q n_e E_\parallel - \nabla_\parallel p_e + \frac{v_e m_e j_\parallel}{e}$$  \hspace{1cm} (2-20)

The electric field is $E_\parallel = -\nabla_\parallel \phi$ and the isothermal pressure gradient is $\nabla_\parallel p_e = T_e \nabla_\parallel n_e$. The temperature gradient is small compared with density gradient due to the fast electron thermal flow associated with $\omega > k_\parallel v_e$. Thus much of the low-frequency drift-wave dynamics falls in the regime where the Boltzmann description of the electron response:

$$\delta n_e = N \left[ \exp \left( \frac{e \phi}{T_e} \right) - 1 \right]$$  \hspace{1cm} (2-21)

applied in the form

$$\delta n_e = N \frac{e \phi}{T_e}$$  \hspace{1cm} (2-22)
Here in the last step it is convenient to adopt an equality in the sense of defining the linear, adiabatic electron model. After dropping the electron inertia ($m_e - 0$) the parallel electron force balance becomes:

$$E_\parallel + \frac{1}{en_e} \nabla_\parallel p_e + \frac{b_e(\nabla \pi_e)}{en_e} = \eta J_\parallel$$  \hspace{1cm} (2-23)

where $\pi_e$ is the traceless momentum stress tensor that includes the electron viscosity and $\eta$ is the electrical resistivity, Horton. (1997), Shenggang. (2002).

2.5.2 Conditions for transport and propagation of disturbances

Now we analyze the ion motion in the $E \times B$ convection. For the small, localized excess of ion charge shown in Fig. (2.4), we get equation as follows:

$$v_E = \frac{cE \times B}{B^2}$$  \hspace{1cm} (2-24)

convection rotates plasma clockwise around the potential maximum $\varphi > 0$ which is also the density and electron pressure maximum in the adiabatic response by

$$\delta n_e \approx N \frac{e\varphi}{T_e}$$

Now, if the ambient plasma is uniform $\partial n_e = \partial T_i = 0$ across the convection zone, then the cell rotates without plasma transport Workshop. et al. (1997), Kaneko, (2001). When the plasma has an $x$ gradient of density (pressure), however, there is a net transport of the structure along the symmetry direction $\hat{y}$ with no net transport across an $x=$ constant surface.
2.6 Physical Mechanism of the Rayleigh-Taylor Instability

The instability of a heavy fluid supported by a light fluid against the gravitational field is a classical problem in hydrodynamics. An analogy is available in the example of an inverted glass of water (Fig. 2.5). Although the plane interface between the water and air is in a state of equilibrium in that the weight of the water is supported by the air pressure, it is an unstable equilibrium. Any ripple in the surface will tend to grow at the expanse of potential energy in the gravitational field. A similar problem arises in a plasma (acts as a heavy fluid) supported by a magnetic field (a light fluid) against gravity or some other force field. Such instabilities are generally categorized as the Rayleigh-Taylor instability. These involving gravity are in particular called the gravitational instability, Chen F.F. (1984), Ichimaru S. (1986).

![Figure 2.5: Hydrodynamic Rayleigh-Taylor Instability of a Heavy Fluid Supported by a Lighter One, Chen, (1984).](image)

To treat the simplest case, we consider a plasma that is only non-uniform in the x direction and is immersed in a uniform magnetic field in the z direction. To be specific, we suppose that the density gradient $\vec{\nabla} n_0$ is in the negative x direction and the gravitational field $\vec{g}$ is opposite to it, i.e. in the positive x direction. This corresponds to the case of a dense plasma supported against gravity by a magnetic field, as shown in (Fig. 2.6). For simplicity we may let $k_B T_e = k_B T_i = 0$, where $k_B$ is Boltzmann’s constant, $T_e$ and $T_i$ are the ion and electron temperatres.

The physical mechanism at work in the Rayleigh-Taylor instability can be understood in terms of the gravitational drifts of the ions and electrons. An external force $\vec{F}$ (such as a gravitational force $\vec{F} = m\vec{g}$) perpendicular to a magnetic field $\vec{B}$
causes a charged particle (in particular, an ion with charge $+e$) to drift with a velocity, Goldston R. J. and Rutherford P. H. (1995).

$$V_0 = \frac{m_i g \mathbf{\hat{x}}}{eB^2} = -\frac{m_i g}{eB} \mathbf{\hat{y}}$$

(2-25)

The electrons have an opposite drift that can be neglected in the limit $\frac{m_e}{m_i} \to 0$

Figure (2.6): Plasma Surface Subject to a Gravitational Instability, Ichimaru S. (1986).

Suppose a small wave like ripple should develop on a plasma-vacuum interface as a result of random fluctuations, as shown in (Fig. 2.7). The gravitational drift of ions on the plasma side of the interface will cause positive charge to build up on one side of the ripple, as illustrated in (Fig. 2.8); the depletion of ions causes a negative charge to build up on other side of the ripple. Due to this separation of charges, a small electric field $\mathbf{E}$ develops, and this electric field changes sign going from crest to trough of the perturbation, again, as shown in (Fig. 2.8). It is apparent that the resulting $\mathbf{E} \times \mathbf{B}$ drift is always upward in those regions where the interface has already moved upward and downward in those regions where the interface has already moved downward. Thus, the initial ripple grows larger, as a result of $\mathbf{E} \times \mathbf{B}$ drifts that are phased so as to amplify the initial perturbation, Goldston R. J. and Rutherford P. H. (1995).
Figure (2.7): Wave-like Perturbation of the Plasma-Vacuum Interface, Goldston R. J. and Rutherford P. H. (1995).

Figure (2.8): The Mechanism of the Rayleigh-Taylor Instability. The Ion Gravitational Drift Leads to Charge Separation on the Plasma-Vacuum Interface, Producing Electric Fields and $\vec{E} \times \vec{B}$ Drifts that Increase the Amplitude of the Perturbation, Goldston R. J. and Rutherford P. H. (1995).
Chapter 3
Theoretical Model
Chapter 3
Theoretical Model

3.1 Introduction

Drift waves are a kind of waves, which exists, in an inhomogeneous plasma. Plasma, especially confined by magnetic field in a laboratory are usually non-uniform with gradients of density, Temperature and so on. Due to the gradients, drift waves are excited universally, Zhang (1992).

In plasma physics. It is fairly common using Magneto Hydro Dynamics equations to describe the properties of plasma. In other words, plasmas are regarded as fluids, where the main reason for this is that the majority of plasma phenomena observed in real experiments can be explained by Magneto Hydro Dynamics theory. In the fluid approximation, we consider that the plasma is composed of two or more interpenetrating fluids. In the simplest case, when there is only one species of ion, we shall need two equations of motion; one is for the positively charged ion fluid and one for the negatively charged electron fluid. The ion and electron fluids will interact with each other even in the absence of collisions, because of the presence of E and B fluids they generate, Zhang (1992).

In the last few years, some theoretical work has been done in cylindrical plasma geometry, which always use the two-fluid equations of motion Egger et al. (1986), Ellis et al. (1980), Evard et al. (1979), Marden-Marshall E. et al. (1986), where some simplifying theoretical assumptions are made. The electron- density distribution is usually taken into account, but the electron-temperature distribution is always assumed to be constant. In fact, however, in a bounded plasma there must be some cooling near the boundary, and hence gradient in temperature should be taken into consideration, Sayasov and Aebischer (1988).

Electron-temperature oscillations are not usually considered. In fact, electron-energy conservation demands that they always exist. The radial electric field is often not considered. This does not make much physical sense, since in cylindrical plasma, the particles diffuse in a direction opposite to the gradient in density. The step length
is the magnitude of the Larmor radius $r_L$ which is the radius of gyration. As a result, the ions move faster than electrons because of their higher Larmor radius, and hence a radial electric field is build up in the direction of density gradient, Chen (1984).

When drift waves occur in a plasma column which has a radial electric field, the drift-wave frequency is affected by plasma column rotation by an $\vec{E} \times \vec{B}$ drift. The frequency of plasma column rotation $\omega_{rot}$ caused by the $\vec{E} \times \vec{B}$ drift is given by Chen (1984), Marden-Marshall et al (1986).

$$\omega_{rot} = \frac{E}{rB}$$

Zhang, (1992), had considered this field, but he eliminated the electron temperature oscillation in his theory.

Electron motion parallel to the magnetic field lines is considered by Marden-Marshall et al. (1986). However, the electron temperature gradient is not included into the theory, which contradicts the real situation.

The physical mechanism at work in the Rayleigh Taylor instability can be understood in terms of the gravitational drifts of the ions and electrons. An external force $\vec{F}$ (such as a gravitational force $\vec{F} = mg$) perpendicular to a magnetic field $\vec{B}$ causes a charged particle (in particular, an ion with charge $+e$) to drift with a velocity, Goldston and Rutherford (1995):

$$V_0 = \frac{m_i g \times \vec{B}}{eB^2} = -\frac{m_i g}{eB} \hat{y}$$

(3-1)

In the present chapter, both electron-density and temperature gradients, electron-temperature oscillations, electron motion parallel to the magnetic field lines and the radial electric field are taken into account. We first formulate our theory independently of any given laboratory plasma, using the full non-viscous electron energy equation and considering the radial dependent of the collision frequencies, making assumptions that are usually well satisfied in the plasma used to study drift waves, so that the theory is valid for a variety of plasmas. We then apply it to specific experimental data (from Egger et al. 1986) and demonstrate its usefulness.
3.2 The Growth Rate of the Instability

In the equilibrium state, the ions obey the equation, Chen (1984):

\[ m_i n_0 (\vec{V}_0 \cdot \vec{\nabla}) \vec{V}_0 = e n_i \vec{V}_0 \times \vec{B}_0 + m_i n_0 \vec{g} \]  
(3-2)

If the streams have the same density and equal but opposite velocities, we may adopt \( V = V_1 \). To find the growth rate, we can perform the usual linearized wave analysis for waves propagating in the y direction (the wave number is \( \vec{k} = k \hat{y} \)).

The perturbed ion equation of motion is, Chen (1984):

\[ m_i (n_0 + n_1) \left( \frac{\partial}{\partial t} (\vec{V}_0 + \vec{V}_1) + (\vec{V}_0 + \vec{V}_1) \cdot \vec{\nabla} (\vec{V}_0 + \vec{V}_1) \right) = e(n_0 + n_1)(E_1 + (\vec{V}_0 + \vec{V}_1) \times \vec{B}_0) + m_i (n_0 + n_1) \vec{g} \]  
(3-3)

Multiply equation (3-2) by \( 1 + \frac{n_1}{n_0} \) to obtain:

\[ m_i (n_0 + n_1)(\vec{V}_0 \cdot \vec{\nabla}) \vec{V}_0 = e(n_0 + n_1)\vec{V}_0 \times \vec{B}_0 + m_i (n_0 + n_1) \vec{g} \]  
(3-4)

Subtracting this from equation (3-3) and neglecting second-order terms, we obtain:

\[ m_i n_0 \left( \frac{\partial \vec{V}_1}{\partial t} + (\vec{V}_0 \cdot \vec{\nabla}) \vec{V}_1 \right) = e n_0 (E_1 + \vec{V}_1 \times \vec{B}_0) \]  
(3-5)

Note that \( \vec{g} \) has cancelled out in term finally of equation (3-4). Information regarding \( \vec{g} \), however, is still contained in \( \vec{V}_0 \). For perturbation of the form \( \exp(ik_y \cdot t) \), we have:

\[ m_i (\omega - k V_0) \vec{V}_1 = ie(E_1 + \vec{V}_1 \times \vec{B}_0) \]  
(3-6)

This equation can be solved and the solution for \( \omega^2_{ci} \gg (\omega - kV_0)^2 \) is as follows:

\[ V_{ix} = \frac{E_y}{B_0} \]  
(3-7)

\[ V_{iy} = -i \frac{E_y}{B_0} \left( \frac{\omega - kV_0}{\omega_{ci}} \right) \]  
(3-8)

where \( \omega_{ci} \) is the ion cyclotron frequency given by \( \frac{eB_0}{m_i} \). The quantity \( V_{ey} \) for electrons vanishes in the limit \( \frac{m_e}{m_i} \to 0 \). For electrons, we therefore have:

\[ V_{ex} = \frac{E_y}{B_0} \]  
and  \[ V_{ey} = 0. \]
the perturbed equation of continuity for ions is given by:

\[
\frac{\partial n_1}{\partial t} + \nabla n_1 \cdot \vec{V}_0 + n_0 \nabla \cdot \vec{V}_1 + \nabla n_0 \cdot \vec{V}_1 = 0
\]  

(3-9)

the zeroth-order term vanishes since \(\vec{V}_0\) is perpendicular to \(\nabla n_0\), and the \(n_1 \nabla \cdot \vec{V}_0\) term vanishes if \(\vec{V}_0\) is constant. therefore, the first order equation is:

\[-i\omega n_1 + ikV_0n_1 + ikV_{iy}n_0 + V_{ix}n_0' = 0\]  

(3-10)

where \(n_0' = \frac{\partial n_0}{\partial x}\). the electrons follow a simpler equation, since \(\vec{V}_{e^*} = 0\) and \(V_{ey} = 0\)

so, we obtain:

\[-i\omega n_1 + V_{ix}n_0' = 0\]  

(3-11)

note that we have used the plasma approximation and have assumed \(n_{i^*} = n_{e^*} = n_1\)

this is possible because the unstable waves are of low frequency \(\omega \ll \omega_{ci}\).

equations (3-7) and (3-10) yield:

\[(\omega - kV_0)n_1 + i \frac{E_y}{B_0} n_0' + ikn_0 \frac{E_y}{B_0} \left( \frac{\omega - kV_0}{\omega_{ci}} \right) = 0\]  

(3-12)

equations (3-8) and (3-11) yield:

\[\omega n_1 + i \frac{E_y}{B_0} n_0' = 0\]  

(3-13)

\[\frac{E_y}{B_0} = \frac{i\omega n_1}{n_0'}\]  

(3-14)

substituting this into equation (3-12), we have:

\[(\omega - kV_0)n_1 - \left\{ n_0' + kn_0 \left( \frac{\omega - kV_0}{\omega_{ci}} \right) \right\} \frac{\omega n_1}{n_0'} = 0\]  

(3-15)

\[(\omega - kV_0) - \left\{ 1 + \frac{kn_0}{\omega_{ci}} \left( \frac{\omega - kV_0}{n_0'} \right) \right\} \omega = 0\]  

(3-16)

\[\omega(\omega - kV_0) = -V_0 \omega_{ci} \frac{n_0'}{n_0}\]  

(3-17)
Substituting for \( V_0 \) from equation (3-1), we obtain a quadratic equation for:

\[
\omega^2 - \omega k V_0 - g \left( \frac{n_0'}{n_0} \right)^1 = 0
\]  

(3-18)

Which has the following solutions

\[
\omega = \frac{1}{2} k V_0 \mp \left[ \frac{1}{4} (k V_0)^2 + g \left( \frac{n_0'}{n_0} \right)^1 \right]^{1/2}
\]

(3-19)

There is an instability if \( \omega \) is complex; that is, if

\[
-g \left( \frac{n_0'}{n_0} \right) > \frac{1}{4} (k V_0)^2
\]

(3-20)

From this, we see that instability requires \( g \) and \( \frac{n_0'}{n_0} \) to have opposite sign. This is just the statement that the light fluid is supporting the heavy fluid; otherwise, \( \omega \) is real and the plasma is stable. For sufficiently small \( k \) (long wavelength), the growth rate is given by;

\[
\gamma = I_m(\omega) \approx \left[ -g \left( \frac{n_0'}{n_0} \right)^1 \right]^{1/2}
\]

(3-21)

The real part of \( \omega \) is \( \frac{1}{2} k V_0 \). Since \( V_0 \) is an ion velocity, this is a low-frequency oscillation, as previously assumed. This instability, which has \( \vec{k} \perp \vec{B}_0 \) is sometimes called a “flute” instability, Chen (1984).

### 3.3 Drift wave theory

The development and analysis of the drift wave theory of collisional drift modes is reported in detail in papers by Ellis and Marden – Marshall (1979), and Ellis et al. (1980).

Therefore, we demonstrate here an outline only of the results of the calculation of the non-local cylindrical model. Two fluid equation, for either species electrons or ions are used to describe the modes. The general form of these is:

\[
m_\xi n_\xi \left\{ \frac{\partial V_\xi}{\partial t} + (\vec{V}_\xi \cdot \nabla) \vec{V}_\xi \right\} = n_\xi e_\xi (\vec{E} + \vec{V}_\xi \times \vec{B}) - \nabla p_\xi - m_\xi n_\xi \nu_\xi \vec{V}_\xi
\]
Consider the ratio of each equation of terms given as:

\[ \frac{\omega}{\omega_{c\xi}} = \left| \frac{m_{\xi} n_{\xi} \omega_{L\xi}}{n_{\xi} e_{\xi} V_{L\xi} B} \right| \tag{3-22} \]

\[ \frac{v_{\xi}}{\omega_{c\xi}} = \left| \frac{m_{\xi} n_{\xi} v_{\xi} V_{L\xi}}{n_{\xi} e_{\xi} V_{L\xi} B} \right| \tag{3-23} \]

The equation of continuity, Dendy (1993):

\[ \frac{\partial n_{\xi}}{\partial t} + \nabla \cdot (n_{\xi} \vec{V}_\xi) = 0 \tag{3-24} \]

The equations of state, for which we take the ideal gas law, Pathria (1972):

\[ P_{\xi} = n_{\xi} K_B T_{\xi} \tag{3-25} \]

The non-viscous electron-energy equation in its reduced form, i.e. the kinetic energy and the Ohmic dissipation terms have been eliminated with the aid of the electron equations of motion and continuity, Braginskii (1965):

\[ \frac{3}{2} n_e \left( \frac{\partial}{\partial t} + V_e \cdot \vec{V} \right) K_B T_e = -\vec{V} \cdot \vec{q}_e - n_e K_B T_e \vec{V} \cdot \vec{V} \tag{3-26} \]

\[ \vec{q}_e \] is the electron thermal flux vector.

The system of equations will be closed with the quasi-electrostatic approximation as

\[ \vec{E} = -\vec{V} \varphi \] and the assumption of quasi-neutrality as \( n_e = n_i \).

Where \( \xi \) is either \( e \) or \( i \) denoting electrons or ions, \( m_{\xi}, e_{\xi}, \text{ and } V_{\xi} \) are the electron (or ion) mass, charge and velocity. Prospectively \( n_{\xi} \) is the charged particle-neutral atom collision frequency. We shall consider a weakly ionized cylindrical low-B plasma coordinates \((r, \theta, z)\) in which collisions between charged particles and neutrals are important but coulomb collisions can be neglected. We consider a column of a plasma confined by a magnetic field in the \( z \)-direction \( \vec{B} = B \hat{z} \). In this
geometry the density n, the pressure P, and the temperature T have gradients perpendicular to the magnetic field are unidirectional (Fig. 3.1), \((\vec{n}, \vec{V}, \vec{T}) \perp \vec{B}\).

Further assumptions are also made, as follows:

1- The plasma has very low \(\beta = nk_bT_e/(B^2/8\pi)\), so that the fluctuations are electrostatic: \(E = -\nabla \varphi\).

2- The ions are relatively cold: \(T_{i0} \ll T_{e0}\). This allows us to neglect the pressure gradient \(\nabla P_i\), and the convective term \((\vec{V}_i \cdot \nabla)\vec{V}_i\) in the ion equation of motion and the terms containing the drift velocity \(\vec{V}_{i0}\) in the ion equation of continuity. However, we take into account the dependence of the ion-neutral collision frequency \(\nu_i\) on \(T_{i0}\). Sayasov and Aebischer, (1988) indicated that \(T_{i0}\) is usually 5-6 times smaller than \(T_{e0}\) in plasmas used to study drift waves so that we considered the ions to be cold \((T_{e0} \approx 0)\).

3- The most important features in identifying the drift instability are the oscillation (fluctuation) level in plasma density, electron temperature and electrostatic
potential. We separate the dependent variables, namely the particle density \( n_\xi \), the fluid velocities \( V_\xi \), the electric potential \( \varphi \), and the electron temperature \( T_e \), into two parts: an “equilibrium” part, indicated by a subscript 0, and a comparatively small oscillating perturbation part indicated by a subscript 1. Considering the propagation properties of drift waves mentioned above, we can represent \( V_\xi \), \( n_\xi \), \( \varphi \) and \( T_e \) in the form Sayasov and Aebischer (1988):

\[
\Psi_\xi = \Psi_{\xi 0} + \Psi_{\xi 1}(r)e^{i(m\theta + k_z z - \omega t)}
\]

(3-27)

where \( \omega \) is the complex drift wave frequency given by

\[
\omega = \omega_R + i\omega_I
\]

(3-28)

4- The plasma may carry a current along \( B_0 \), which can be represented by a drift of the electrons at speed \( u_0 \) in the z-direction.

5- The plasma is quasi-neutral at all times: \( n_e = n_i \). The condition required for this of the plasma is approximation to be valid that the Debye length \( \lambda_D = (\varepsilon_0 k_B T_e / n_0 e^2)^{1/2} \) Small compared with the physical size of the plasma \( L (\lambda_D \ll L) \), Sturrock (1994).

6- This assumption implies that \( n_{e 0} = n_{i 0} = n_0 \) and \( n_{e 1} = n_{i 1} = n_1 \) at all times, Zhang (1992).

7- The equilibrium density, \( n \), varies radially across the field, and there is no equilibrium electric field perpendicular to \( \vec{B} \).

8- The plasma may carry a current along \( B \), which is represented by drift of the electrons at speed \( u \) in the z-direction, Zhang (1992).

9- The phase velocity of the drift wave parallel to \( B_0 \) is much higher than the ion thermal velocity \( \omega / k_z \gg (k_B T_i / m_i)^{1/2} \). This allows us to neglect ion motion parallel to \( \vec{B}_0 \), and can not used group velocity \( \frac{d\omega}{dk_z} \).
3.4 Basic Equations

The two-fluid equations ($\xi = e, i$) are as follows

The equation of motions is given by, Self (1970):

$$m_\xi n_\xi \left\{ \frac{\partial \vec{V}_\xi}{\partial t} + (\vec{V}_\xi \cdot \vec{\nabla}) \vec{V}_\xi \right\} = n_\xi e_\xi (\vec{E} + \vec{V}_\xi \times \vec{B}_0) - \vec{\nabla} p_\xi - m_\xi n_\xi v_\xi \vec{V}_\xi$$

(3-29)

Where collisions are represented by the drag term $m_\xi n_\xi v_\xi \vec{V}_\xi$.

Drift waves are low-frequency electron and ion-density oscillations propagating azimuthally with the well-known diamagnetic drift velocity. They are characterized by their azimuthal mode number $m$ (number of cycles occurring azimuthally). Usually they are observed to propagate also axially. Drift-wave instabilities are driven by the radial pressure gradient in the plasma, Chen (1984).

Equations (3-26) in the $z$-direction $\vec{B} = B\hat{z}$ can easily be solved by the procedure of linearization. By this we assume that the amplitude of oscillation is small. Thus, terms with higher power of amplitude factors can be neglected Chen (1984). The most important features in identifying the drift instability are the oscillation (fluctuation) level in plasma density, electron temperature and electrostatic potential. We separate the dependent variables, namely the particle density $n_\xi$, the fluid velocities $V_\xi$, the electric potential $\varphi$, and the electron temperature $T_e$, into two parts: an “equilibrium” part, indicated by a subscript 0, and a comparatively small oscillating perturbation part indicated by a subscript 1. Considering the propagation properties of drift waves mentioned above, we can represent $V_\xi, n_\xi, \varphi$ and $T_e$ in the form Sayasov and Aebischer, (1988):

$$\Psi_\xi = \Psi_\xi^0 + \Psi_\xi^1(r)e^{i(m\theta + k_z z - \omega t)}$$

(3-29’)

Where $\omega$ is the complex drift wave frequency which is given by:

$$\omega = \omega_R + i\omega_I$$

(3-30)
Where $\omega_R$ is the real part of $\omega$, and imaginary part $\omega_I$ represents the growth rate of the corresponding drift wave mode $m$.

Applying the general set of equations (3-29’)-(3-26) to drift waves, and adopting the following assumptions as described in different literatures Marden-Marshall et al. (1986), Politzer (1971), Zhang (1992). We have this assumption further allows us to assume that the electron diamagnetic velocity $\vec{V}_{e0}$ has a component in the $\theta$-direction only. It follows from the equilibrium electron equation of motion that,

\[
\vec{V}_{de} = -\frac{K_B T_e}{eB_0} \left( \frac{1}{T_e} \frac{dT_e}{dr} + \frac{1}{n_{e0}} \frac{dn_{e0}}{dr} \right) \hat{\theta}
\]

\[
\vec{V}_{e0} = -\frac{K_B T_e}{eB_0} \left( \frac{1}{T_e} \frac{dT_e}{dr} + \frac{1}{n_{e0}} \frac{dn_{e0}}{dr} \right) \hat{\theta}
\]

(3-31)

The temperature-gradient term enters naturally when $T_e$ is allowed to vary radially in a magnetized plasma. For simplicity, let us introduce the following shorthand notation:

\[
\overrightarrow{K} = \frac{1}{n_{e0}} \frac{dn_{e0}}{dr} \hat{\hat{r}} \quad , \quad \overrightarrow{K'} = \frac{1}{T_e} \frac{dT_e}{dr} \hat{\hat{r}}
\]

So equation (3-31) becomes:

\[
\vec{V}_{e0} = -\frac{K_B T_e}{eB_0} (K' + K) \hat{\theta}
\]

(3-32)

The thermal flux vector $q_e$ can also be assumed to be parallel to the magnetic field, and is given by Braginskii (1965):

\[
\vec{q}_e = -\lambda_z \frac{\partial T_e}{\partial z} \hat{\hat{z}}
\]

(3-33)

where $\lambda_z$ is the thermal conductivity along the magnetic field $B_0$. 

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We assume a sheared radial electric potential $\varphi_0(r)$ i.e. an electric field $E$ in the direction of $\vec{V} \times \vec{B}$ drift velocity $\vec{V}_E$.

\[
\vec{V}_E = \frac{\vec{E} \times \vec{B}_0}{B_0^2} = \frac{E}{B_0} \hat{\theta}
\]  

(3-34)

We obtain the following set of linearized equations from the general hydrodynamical equations (3-29)-(3-26).

The ion equation of motion:

\[
-i \omega m_i n_i^0 \vec{V}_{i1} = -e n_i^0 \vec{V}_0 n_{i1} - e n_i^1 \vec{V}_0 n_{01} + e n_i^1 B_0 \vec{V}_{i1} \times \hat{Z} + en_i^1 \vec{V}_E B_0 \hat{r} - im_i n_i^0 \vec{V}_{i1}
\]  

(3-35)

The ion equation of continuity:

\[
-i \omega n_i^1 + \frac{1}{r} \frac{\partial n_i^1}{\partial \theta} V_E + \vec{V}_0 n_{i1} + n_i^0 \vec{V}_0 n_{i1} = 0
\]  

(3-36)

The electron equation of motion perpendicular to the magnetic field lines:

\[
0 = -k_B n_e \vec{V}_0 ^0/e_0 - k_B T_e ^0 \vec{V}_0 n_{e1} - k_B \vec{V}_0 \vec{V}_0 ^0 n_0 + en_e^0 \vec{V}_0 n_0 + en_e^0 \vec{V}_0 \varphi_1 - en_i^0 B_0 \vec{V}_{i1} \times \hat{Z} - en_i^0 B_0 (V_E + V_{de}) \hat{r}
\]  

(3-37)

The electron parallel equation of motion:

\[
0 = i k_z (-k_B T_e ^0 n_{e0} - k_B T_e ^0 n_{e1} + en_e^0 \varphi_1) - m_e v_e n_0 V_{e1z}
\]  

(3-38)

The electron equation of continuity:

\[
-i(\omega - \omega_1) n_{e1} + (V_E + V_{de}) \frac{\partial n_{e1}}{\partial \theta} + V_e^{1r} \frac{\partial n_{e0}}{\partial r} + n_{e0} \vec{V} \cdot \vec{V}_{e1} = 0
\]  

(3-39)

The full non-viscous electron-energy equation:

\[
\frac{3}{2} n_{e0} \left[ k_B T_e ^0 V_e^{1r} K' + i \left( V_{de} \frac{m}{r} - (\omega - \omega_1 - \omega_E) \right) k_B T_e ^0 \right] = -\lambda_z k_z^2 k_B T_e ^0 - i k_B T_e ^0 (\omega - \omega_1 - \omega_E) n_{e1} + k_B T_e ^0 (i V_{de} \frac{m}{r} n_{e1} + n_{e0} V_{e1r} K)
\]  

(3-40)

Where:

\[
\omega_1 = k_z u_0
\]  

(3-41)
and

\[ \omega_E = k_\theta V_E = \frac{m}{r} E = \frac{m}{r} \frac{1}{B_0} \frac{d\varphi_0}{dr} \quad (3-42) \]

Represent the drift wave frequency due to the electron motion parallel \( B_0 \) and \( \vec{E} \times \vec{B} \) rotation respectively.

It is possible to reduce the above system of equations to one single ordinary differential equation for the oscillating potential \( \varphi_1 \) by the procedure described below.

The intermediate formulas that will be obtained are useful to describe the drift wave phenomena and to reveal important relations between the physical quantities.

From the linearized ion equation of motion (3-35), we can express the oscillating ion velocity \( \vec{V}_{i1} \) in terms of the oscillating potential \( \varphi_1 \):

\[ \vec{V}_{i1} = \frac{1}{B_0} \left[ i \frac{\omega + i v_i}{\omega_{ci}} \left( \vec{\nabla} \varphi_1 + \frac{n_{i1}}{n_{i0}} \vec{\nabla} \varphi_0 - B_0 \frac{n_{i1}}{n_{i0}} V_E \hat{r} \right) + \hat{z} \times \vec{\nabla} \varphi_1 + \frac{n_{i1}}{n_{i0}} \hat{z} \times \vec{\nabla} \varphi_0 \right] \]

\[ \quad \vec{V}_0 - B_0 \frac{n_{i1}}{n_{i0}} V_E \hat{\theta} \quad (3-43) \]

Substituting this expression into the ion equation of continuity (3-36), yields the ion density oscillation \( n_{i1} \) in terms of \( \varphi_1 \):

\[ \frac{n_{i1}}{n_{i0}} = \frac{e}{k_B T_{e0}} \left[ \frac{\omega + i v_i}{(\omega - \omega_E)} \rho^2 \left( \frac{\partial^2 \varphi_1}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi_1}{\partial r} - \frac{m^2}{r^2} \varphi_1 + K \frac{\partial \varphi_1}{\partial r} + \omega_{de} \frac{\partial \varphi_1}{(\omega - \omega_E)} \right) \right] \quad (3-44) \]

Where:

\[ \nabla^2 \perp = \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{m^2}{r^2} \]

Where the electron temperature \( T_{e0} \) in the numerator instead of the ion temperature \( T_{i0} \) Sayasov and Aebischer (1988):

\[ \rho^2 = \frac{K B T_e}{m_i \omega^2} \quad (3-45) \]
We further define:

\[ \omega'_{de} = - \frac{K_B T_{e0}}{eB_0} m \frac{K'}{r}, \quad \omega_{de} = - \frac{K_B T_{e0}}{eB_0} m \frac{K}{r} \]  

(3-46)

The procedure for the electron equations is more involved. First, the electron perpendicular equation of motion (3-37) can be solved for the components \( V_{e1r} \hat{r} \) and \( V_{e1\theta} \hat{\theta} \). The electron parallel equation of motion (3-38) can also be solved for the component \( V_{e1z} \) of the oscillating electron velocity \( V_{e1r} \). This yields

\[
V_{e1r} = -i \frac{K_B T_{e0}}{eB_0} m \left( \frac{e\phi_1}{K_B T_{e0}} - \frac{n_{e1}}{n_{e0}} - \frac{T_{e1}}{T_{e0}} \right) 
\]

(3-47)

\[
V_{e1r} = \frac{1}{eB_0 n_{e0}} (k_B \left( -T_{e0} \frac{\partial n_{e1}}{\partial r} - n_{e1} \frac{d T_{e0}}{d r} - T_{e1} \frac{d n_{e0}}{d r} - n_{e0} \frac{\partial T_{e1}}{\partial r} \right)) + e n_{e1} \frac{d \phi_1}{d r} + e n_{e0} \frac{d \phi_1}{d r} - eB_0 n_{e1}(V_{de} + V_E) 
\]

(3-48)

\[
V_{e1z} = -i \frac{K_z}{m_e v_e} \left( k_B T_{e0} \frac{n_{e1}}{n_{e0}} + k_B T_{e1} - e \phi_1 \right) 
\]

(3-49)

The above equations can then be substituted into the electron equation of continuity (3-39). After a mathematical treatments, we have obtained a simple form for the electron equation of continuity in terms of \( \phi_1, n_{e1} \), and the electron-temperature oscillation \( T_{e1} \):

\[- i(\omega - \omega_1 - \omega_E + iv_\parallel) n_{e1} + \frac{v_{\parallel}/n_{e0}}{k_B T_{e1}} \left( k_B T_{e1} - e \phi_1 \right) - i \frac{n_{e0}}{r} K \phi_1 = 0 \]

(3-50)

Where \( v_\parallel \) is a shorthand notation for the expression:

\[ v_\parallel = \frac{K_z^2 k_B T_{e0}}{m_e v_e} \]  

(3-51)

We then continue by substituting the expression (3-47) for \( V_{e1r} \hat{r} \) into the electro energy equation (3-40) and solving for the electron-temperature oscillation \( T_{e1} \). We get:

\[
T_{e1} = \frac{i T_{e0} \left( (\omega_{de} - \frac{3}{2} \omega')_{de} \right)^{\frac{e \phi_1}{K_B T_{e0}} + \frac{5}{2} \omega'_{de} - \omega_s} \frac{n_{e1}}{n_{e0}})}{(\omega_2 + \frac{1}{2} (5 \omega_{de} - 3 \omega_s))} 
\]

(3-52)
Where

\[ \omega_s = \omega - \omega_1 - \omega_E \]  
(3-53)

\[ \omega_z = \frac{1}{n_0} \lambda_z K_Z^2 \]  
(3-54)

If we then substitute equation (3-52) for \( T_{e1} \) into the equation of continuity (3-50), the electron density oscillation in terms of \( \varphi_1 \) can be produced, as we did for the ions in (3-44):

\[
\frac{n_{e1}}{n_{e0}} = \frac{(v_j - i \omega_{de}) \left[ \omega_z + i \frac{3}{2} (\omega_{de}^* - \omega_s) + \omega_{de} (\omega_{de} - \frac{3}{2} \omega'_{de}) \right] + e \varphi_1}{(v_j - i \omega_s) \left[ \omega_z + i \frac{3}{2} (\omega_{de}^* - \omega_s) + \omega_s (\omega_{de} - \frac{3}{2} \omega_{de}) + i v_j (\omega_{de}^* - \omega_s) \right] K_B T_{e0}}
\]  
(3-55)

Where

\[ \omega_{de}^* = \omega_{de} + \omega'_{de} \]  
(3-56)

Clearly, this perturbed electron density is similar to the form of the perturbed ion density that is given in equation (3-44).
Chapter 4
Theoretical Treatment and Results
Chapter 4  
Theoretical Treatment and Results  

4.1 Second Order Differential Equation for Drift Waves  

Theoretical consideration of a drift wave in a collisionless plasma within a uniform magnetic field is given in the present work. Linearisation of the basic equation for both fluids, and using assumptions (1) to (9) result in the differential equation form for $\varphi_1$ derived by Ubeid M. (2002):

$$\frac{\partial^2 \varphi_1(r)}{\partial r^2} + \left[ \frac{1}{r} + K(r) \right] \frac{\partial \varphi_1(r)}{\partial r} + \left[ Q(r, \omega) - \frac{m^2}{r^2} \right] \varphi_1(r) = 0 \quad (4-1)$$

Where

$$Q(r, \omega) = \frac{\omega - \omega_E}{p^2(\omega + ivl)} \times \left\{ \omega_{de} \left( n_{e} \right) \left( \omega + \frac{3}{2} \omega_{de}^{*} - \omega_{s} \right) + \omega_{de} \left( \omega_{de} - \frac{3}{2} \omega_{de}^{*} \right) \right\}$$

With boundary conditions at the plasma beam center $r = 0$, and the plasma beam radius $r = r_0$, Marden-Marshall et al. (1986), Sayasov and Aebischer (1988):

$$\varphi_1(0) = 0 \quad , \quad \varphi_1(r_0) = 0 \quad (4-3)$$

The difference between equation (4-1) and the equation obtained by Sayasov and Aebischer, (1988) is the $Q(r, \omega)$ value.

Then, equation (4-2) becomes similar to the expression derived by Ellis, et al. (1980). Equations (4-1)-(4-3) represent a complex-eigenvalue problem for the complex drift-wave frequency $\omega$ and the complex eigenfunction $\varphi_1(r)$, the radial distribution of the oscillating electric potential. It can be solved numerically, especially for arbitrary given undistributed density and temperature profiles $n_0 (r)$ and $T_{e0} (r)$ and the $E \times B$ rotation terms. Once $\varphi_1(r)$ is known, the remaining oscillating quantities can be computed with the aid of (3-43)-(3-56) In addition, the maximum wave amplitude position of the drift wave can be determined. This would show where the drift wave localize in the plasma beam region.
4.2 Doppler Effect on $\omega_r$ measurement

In the observation of drift waves, the frequencies of the drift wave's oscillations were able to obtain. But, the frequencies could not be compared directly with theory since the plasma column was rotating. In order to check the experimental $\omega_r$ with theoretical $\omega_r$ values, the Doppler Effect arising form the plasma column rotation must be taken into account. In laboratory plasma column, a radial electric field usually exists. The electric field $E_r$ originates from radial profiles of the probe floating potential and the electron temperature. Meanwhile, there is a constant magnetic field $B_z$ along the axis. It is determined by $E_r$ and $B_z$ that the plasma column must be rotating and the rotating frequency is, Zhang (1992):

$$\omega_{rot} = \frac{E}{rB}$$

Marshall and Ellis, (1985) state that this rotation is nonuniform, or sheared, where B is constant and E does not very linearly with radius. In most instances, column rotation is assumed uniform. Also, it is used to Doppler shift the theoretical real frequency of the azimuthally propagation drift wave for comparison with experiment. Zhang (1992).

Theoretical consideration of the plasma column rotation can be seen in Marshall and Ellis (1985), there the quantitively analysis was shown by the equation Zhang (1992):

$$\nabla_\perp^2 + \frac{1}{n} \frac{dn}{dr} \frac{d}{dr} + \left[ 1 + \frac{2\omega_\phi}{m\omega_i} - \frac{\omega_\phi^2}{m\omega_i(\omega - \omega_\varphi + iv_i)} \right] Q (\omega - \omega_\varphi) -$$

$$\frac{(\omega^* - iv_i)(2\omega_\phi\omega - \omega_\varphi^2)}{p^2m\omega_i(\omega - \omega_\varphi + iv_i)(\omega - \omega_\varphi - \omega_i + iv_i)} \psi = 0$$

(4-4)

We get on equation (4-4):

$$\frac{\partial^2 \varphi_1 (r)}{\partial r^2} + \left[ \frac{1}{r} + K(r) \right] \frac{\partial \varphi_1 (r)}{\partial r} + \left[ 1 + \frac{2\omega_\phi}{m\Omega_i} - \frac{\omega_\phi^2}{m\Omega_i(\omega - \omega_\varphi + iv_i)} \right] Q (\omega - \omega_\varphi) -$$

$$\frac{(\omega^* - iv_i)(2\omega_\phi\omega - \omega_\varphi^2)}{p^2m\Omega_i(\omega - \omega_\varphi + iv_i)(\omega - \omega_\varphi - \omega_i + iv_i)} \frac{m^2}{r^2} \varphi_1 (r) = 0$$

(4-5)
where:

$$\nabla^2 \perp = \frac{d}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{m^2}{r^2}$$

(4-6)

$$Q(\omega - \omega_\phi) = \frac{iv_1(\omega^* - \omega + \omega_\phi) - \omega^* \omega_\parallel}{p^2 m\omega_i(\omega - \omega_\phi + iv_1)(\omega - \omega_\phi - \omega_\parallel + iv_1)}$$

(4-7)

$$l = \left[ 1 + \frac{2\omega_\phi}{m\Omega_i} - \frac{\omega_\phi^2}{m\Omega_i(\omega - \omega_\phi + iv_1)} \right]$$

(4-8)

$$g = \frac{(\omega^* - iv_1)(2\omega_\phi\omega - \omega_\phi^2)}{p^2 m\Omega_i(\omega - \omega_\phi + iv_1)(\omega - \omega_\phi - \omega_\parallel + iv_1)}$$

(4-9)

G and L this is shortcut to facilitate the equation.

Where in, Zhang (1992):

$$\omega^* = k_\theta \vec{V}_{de} = -\frac{K_BT_{e0}}{eB_0} \left( \frac{1}{n_{e0}} \frac{dn_{e0}}{dr} \right)$$

(4-10)

$$\omega^* = \frac{\Omega}{1 + r^2/a^2}$$

(4-11)

$$\Omega = \frac{2mT}{Ba^2} = \omega^*(0)$$

(4-12)

And

$$\omega_\phi = k_\theta V_{\phi} = \frac{mE}{rB_0} = \frac{m}{rB_0} \frac{1}{dr} \frac{d\phi_0}{dr}$$

(4-13)

We get the general equation of states, we get:

$$\left[ \nabla^2 \perp + \frac{1}{n} \frac{dn}{dr} \frac{d}{dr} + lQ(\omega - \omega_\phi) - g \right] \Psi = 0$$

(4-14)

$$\frac{\partial^2 \phi_1(r)}{\partial r^2} + \left[ \frac{1}{r} + K(r) \right] \frac{\partial \phi_1(r)}{\partial r} + \left[ lQ(\omega - \omega_\phi) - g - \frac{m^2}{r^2} \right] \phi_1(r) = 0$$

(4-15)

With boundary conditions at the plasma beam center r = 0, and the plasma beam radius r = r_0

$$\phi_1(0) = 0 \quad , \quad \phi_1(r_0) = 0$$
Which is the expression obtained by Sayasov and Aebischer, (1988). On other hand, if the radial electron temperature is also considered constant, i.e.

\[
\frac{dT_e^0}{dr} = 0 \quad \text{and} \quad \omega_{de} = \omega_{de}^*
\]  

Equation (4-5) is a complex, second-order differential equation with \( \varphi \) and \( \omega \) complex and \( \omega_r \) and \( \omega_i \) serving as the eigenvalues. If the rotation is neglected together we and take data from Zhang (1992), then all:

\[
\left[ \nabla^2_\perp + \frac{1}{n} \frac{dn}{dr} \frac{d}{dr} + Q(\omega) \right] \varphi = 0 \]  

(4-17)

\[
Q(\omega) = \frac{iv_\| (\omega^* - \omega) - \omega^* \omega_\|}{p^2 m \omega (\omega + iv_\|)(\omega - \omega_\| + iv_\|)}
\]  

(4-18)

\[
\frac{\partial^2 \varphi_1(r)}{\partial r^2} + \left[ \frac{1}{r} + K(r) \right] \frac{\partial \varphi_1(r)}{\partial r} + \left[ Q(\omega) - \frac{m^2}{r^2} \right] \varphi_1(r) = 0
\]  

(4-19)

Now we can realize the equation (4-17) is same as equation (4-1). The value of \( \omega_r \) from equation (4-5). Is much closer to the measured \( \omega_r \) than the value predicted by equation (4-17), Zhang (1992).

Therefore, the \( \vec{E} \times \vec{B} \) rotation of the plasma column is an important factor in governing the stability and frequency of drift waves. A value of \( \omega \) should be found and added to the theoretical real frequency as a Doppler shift when we make a comparison between the experimental and theoretical results, Zhang (1992).

4.3 Numerical Analysis

In this chapter, an illustrative theoretical prediction is given for a magnetized plasma where the drift wave theory simplified to the collisionless plasma regime. The principle motivation for the interpretation of the numerical result is the comparison of theory and experiment. The majority of the results presented were made with the aid of a computer using Matlab integration method- Radial plasma density and electron temperature are also obtained and used as input to a theoretical model. In the present work we have focused mainly on the collisionless plasma regime magnetized into an axial magnetic field. The density gradient scale length is
the important factor for a drift wave, where the absolute value of the density is less significant.

Equation (4-1) is a second order differential equation, which predicts an eigenvalue problem for the complex drift-wave frequency $\omega$ and the eigenfunction $\varphi_1(r)$, which represents the radial distribution of the oscillating electric potential. The boundary conditions take into consideration that $\varphi_1(r)$ vanishes at both the plasma beam center and edge. One suitable general strategy for numerical solution of an eigenvalue problem is an iterative one. We guess a trial eigenvalue and generate a solution by integrating the differential equation as an initial value problem. If the resulting solution does not satisfy the boundary conditions, we change the trial eigenvalue and integrate again, repeating the process until a trial eigenvalue is found for which the boundary conditions are satisfied, such that if one integrates the differential equation (4-1), starting at one boundary and considering the boundary condition there, the resulting solution $\varphi_1(r)$ automatically satisfies the condition at the other boundary. The boundary value problem is thus transformed into an initial-value problem. This can be solved with the aid of the Matlab integration method if one transforms the original complex second-order equation (4-1) into a set of coupled first-order equations.

Once the eigenvalue $\omega$ is computed, the eigenfunction $\varphi_1(r)$ can be found by Matlab integration as described above. The boundary condition on the boundary into which one integrates will then be automatically satisfied, since the proper value $\omega$ is now used. With the aid of the equations given the radial distributions of all other oscillating quantities can then be calculated from $\varphi_1(r)$.

4.4 Numerical Results

In the theoretical area the isothermal model is simplified to a collisionless plasma regime. This model predicts the actual frequency and the radial fluctuation profiles, including the position of maximum wave amplitude. The theoretical values are obtained by using the experimental data of radial density and electron temperature profiles as an input into the theoretical model. The effect of the electron-temperature
is proved to be important for collisional plasma regime. According to the following example when the electron temperature is taken into account the theory gives:

\[ k_B T_{e0} = 3.5eV, \quad B_0 = 0.77T, \quad r_0 = 2.8cm, \quad v_{li} = 7.7 \times 10^5, \quad \omega_z = 1.7v_{li}, \quad v_i = 2.1 \times 10^5. \]

The drift wave frequency for \( m=6 \) mode is: \( \omega_R = 3.5 \times 10^5s^{-1} \) and the growth rate: \( \omega_I = 1 \times 10^4s^{-1} \), \( \omega^* = 60 \times 10^3s^{-1} \), considering radial distribution of density profile is fitted quite well and the measured radial number of density profile can be approximated by the relation:

\[
\frac{n(r)}{n_0} = \frac{1}{(1+r^2/a^2)} \tag{4-20}
\]

where \( a \), plasma beam radius is constant depends on the magnetic field and the pressure. The fitting curve for the measured electron temperature is also approximated as:

\[
\frac{T_e(r)}{T_{e0}} = \frac{1}{(1+r^2/c^2)} \tag{4-21}
\]

where \( c \) is similar to, but smaller than \( a \) (\( a = 7mm, \ c = 4mm \)).

So:

\[
\frac{dn}{dr} = -\frac{2r}{a^2} \frac{n_0}{(1+r^2/a^2)^2} \tag{4-22}
\]

or

\[
\frac{1}{rn_0} \frac{dn}{dr} = -\frac{2}{a^2} \frac{1}{(1+r^2/a^2)^2} \tag{4-23}
\]

In such circumstances, equation (4-1) and (4-15) becomes:

\[
\frac{\partial^2 \varphi_1(u)}{\partial u^2} + \left[ \frac{1}{u} - \frac{2u}{1+u^2} \right] \frac{\partial \varphi_1(u)}{\partial u} + \left[ Q(u, \omega) - \frac{m^2}{u^2} \right] \varphi_1(u) = 0 \tag{4-24}
\]

\[
\frac{\partial \varphi_1(u)}{\partial u} + \left[ \frac{1}{u} - \frac{2u}{1+u^2} \right] \frac{\partial \varphi_1(u)}{\partial u} + \left[ IQ(\omega - \omega_\phi) - \frac{m^2}{r^2} \right] \varphi_1(u) = 0 \tag{4-25}
\]

\[
\frac{\partial^2 \varphi_1(u)}{\partial u^2} + \left[ \frac{1}{u} - \frac{2u}{1+u^2} \right] \frac{\partial \varphi_1(u)}{\partial u} + \left[ Q(\omega) - \frac{m^2}{u^2} \right] \varphi_1(u) = 0 \tag{4-26}
\]
\[ u = \frac{r}{a} \quad (4-27) \]

where m mode numbers of values 1, 2, 3, 4, 5

and \( Q(u, \omega) \) is the same as in equation (4-1), and \( Q(\omega) \) is similar as in equation (4-18), but \( r \) is replaced by \( u \) and 

\[ 1/\rho^2 \text{ by A, where A is:} \]

\[ \begin{align*}
A &= \frac{a^2}{\rho^2} \\
\rho^2 &= 4 \times 10^{-8} \left( \frac{eB_0}{kB_T e^2} \right) \\
\end{align*} \quad (4-28) \]

Typical radial profile of electron density is shown in figure (4.1). The presented data is for an axial magnetic field 875 G, whereas the density level is found to decrease significantly in the plasma beam center but the same level at the edge. The density shows a gradient with the same general trend. Such a strong gradients of radial density tend to drive drift wave of plasma instability.

The radial variation of the electron temperature is also illustrated in figure (4-2). It is found that the electron temperature is of relevance to the identification of drift waves. The electron temperature exhibits radial gradients slightly smaller than the density gradient. The potential profiles in a magnetized plasma also show an overall increase value with the distance from the beam center, indicating that a radial electric field exists directed inwardly. This produces an \( \vec{E} \times \vec{B} \) azimuthal rotation of the plasma which contributes to the plasma drift wave frequency.

Figure (4.3) illustrates the relationship between the density gradient scale length as described with \( \frac{n_1}{n_0} = \frac{1}{(1+r^2/a^2)} \) profiles and plasma beam radius \( r(mm) \), at a magnetic field \( B = 875 \, G \).
Figure (4.1): The relationship between the density $n_0 = 2.8 \times 10^{14} m^{-3}$ profiles at magnetic field and plasma beam radius $r (mm)$, and this curve is drawn using published experimently data, with a magnetic field $B = 875 \ G$.

Figure (4.2): The variation in the relationship between the electron temperature $T_e(0) = 12 eV$ profiles at magnetic field and plasma beam radius $r (mm)$, and this curve is drawn using as above results, with a magnetic field $B = 875 \ G$. 
The relationship between the density obtained by the equation \( \frac{n_1}{n_0} = \frac{1}{(1+r^2/a^2)} \) are used, with a magnetic field \( B = 875 \, G \).

To see how the variation of the electron temperature affects the drift wave characteristics, the radial wave equation (4-24) was solved by ignoring the \( \vec{E} \times \vec{B} \) rotation terms (i.e. the equation that obtained by Aebischer and Sayasov, 1988). To demonstrate that the radial temperature profile \( T_e (r) \) influences the radial drift-wave amplitude distribution, the radial wave equation was also solved for a hypothetical plasma column which has the same density profile as the typical case, but a constant electron temperature in radius (i.e. the equation that obtained by Ellis et al. 1980). In this process for the hypothetical plasma with \( T_e \) is constant (equation of Ellis) the eigen-frequency for \( m = 6 \) mode is \( 3.3 \times 10^5 \, s^{-1} \) is purely real. For the typical plasma in which the radial variation of \( T_e \) is taken into account we find that the eigen frequency for \( m = 6 \) mode is: \( \omega_r = 4.3 \times 10^5 \, s^{-1} \), \( \omega_i = 0.86 \times 10^4 \, s^{-1} \) i.e. \( \omega_i > 0 \) This proves the importance of the effect of the electron-temperature variation and the usefulness of the theory of Aebischer and Sayasov, (1988). The two eigen-functions for the typical and the hypothetical plasmas are displayed in figure (4.4). The eigen function \( \varphi_1 (u) \) profiles in arbitrary units (a.u.) obtained from the
theory for m = 6 mode, with the $\vec{E} \times \vec{B}$ rotation for hypothetical plasma with a constant radial variation of the electron temperature is illustrated in Figure (4.4.a). The maximum value at 2.39 and Figure (4.4.b) with the $\vec{E} \times \vec{B}$ rotation; for real laboratory plasma with the electron Temperature Variation where the maximum value for real laboratory plasma equals 2.63 and Figure (4.4.c) with the $\vec{E} \times \vec{B}$ rotation in addition to the radial variation of the electron temperature where the maximum value with $\vec{E} \times \vec{B}$ plasma equals 2.9, Note the effect $\vec{E} \times \vec{B}$ of increasing the maximum values examine values The eigen function $\varphi_1(u)$. 

In the present work the electric field is included by considering a sheared electric potential given by $\varphi_0 (r) = b_1 + b_2 r^2$, where $b_1$ and $b_2$ are constants determined by measuring the equilibrium potential at the boundary $r = 0$, $r = r_0$ Marden-Marshall and Hall, (1986).

This potential produces a sheared electric field $\vec{E}(r)$ and hence a non-sheared rotation frequency $\omega_E$. If $\vec{E} = 0$ then $\omega_E = 0$ and the eigenfunction $\varphi_1(r)$ and the eigenvalue $\omega$ become as those obtained by Sayasov and Aebischer, (1988).

However, in the present work, when the potential is considered and gives rose a radial electric field $\vec{E}(r)$ so as $\vec{E} \times \vec{B}$ rotation occurs. A calculation is made for an $\vec{E} = .254 \text{ Vm}^{-1}$, and a magnetic field $B = 875 \text{ G}$ then, $\omega_E$ is obtained theoretically and gives $\omega_E = 5.8 \times 10^3 \text{ s}^{-1}$. Also, the eigenfunction $\varphi_1(r)$ and the eigenvalue $\omega$ become as those obtained by Zhang, (1992).
Figure (4.4.a): The Eigen Function $\varphi_1(u)$ in arbitrary units (a.u.) for $m = 6$ mode, with the $\vec{E} \times \vec{B}$ rotation for hypothetical plasma with a constant radial variation of the electron temperature.

Figure (4.4.b): The Eigen Function $\varphi_1(u)$ in arbitrary units (a.u.) for $m = 6$ mode, with the $\vec{E} \times \vec{B}$ rotation for real laboratory plasma with the electron temperature Variation.

Figure (4.4.c): The Eigen Function $\varphi_1(u)$ in arbitrary units (a.u.) for $m = 6$ mode, with the $\vec{E} \times \vec{B}$ rotation in addition to the radial variation of the electron temperature.
Figure (4.5): The radial eigen function $\varphi_1(u)$ obtained theoretically and the predicted experimentally as well as with $\vec{E} \times \vec{B}$ rotation included into a theoretical model, Egger. (1986), present work.

Figure (4.5) difference maximum values the eigen function $\varphi_1(u)$ in arbitrary units (a.u.) for $m = 6$ mode, for the three cases described above., and a theoretical plasma is the maximum value for theoretical plasma equal 2.3 and the maximum value for real laboratory plasma equal 2.6 and the maximum value with $\vec{E} \times \vec{B}$ rotation plasma equal 2.9. Note the effect $\vec{E} \times \vec{B}$ of increasing the maximum values examine values the eigen function $\varphi_1(u)$.

Figure (4.6) displays the relationship between the eigen function $\varphi$ for $m = 1$ profiles at magnetic field and plasma beam radius $r(mm)$, and this curve is drawn using the theoretically normalised results, with a magnetic field $B = 875 \, G$, with sheared $\vec{E} \times \vec{B}$ rotation. The maximum value for the eigen function $\varphi$ is at $r = 3.3 \, mm$ that indicates the impact of the sheared $\vec{E} \times \vec{B}$ rotation.
Figure (4.6): The relationship between the radial eigen function $\varphi$ for $m = 1$ profiles predicted from a theoretical model and plasma beam radius $r(mm)$ in equation (4-26), with a magnetic field $B = 875 \ G$.

Figure (4.7) exhibits the relationship between the eigen function $\varphi$ obtained theoretically for $m = 1,2,3,4$ and 5 profiles at magnetic field and plasma beam radius $r(mm)$. This figure illustrates the effect of sheared $\vec{E} \times \vec{B}$ rotation on the drift wave, when the maximum value for the eigen function $\varphi$ for $m = 1$ with no $\vec{E} \times \vec{B}$ rotation is found at $r = 2.9 \ mm$. However, when sheared $\vec{E} \times \vec{B}$ is considered the maximum value is found at $r = 3.3 \ mm$. That is the mode maximum moves a little further from the beam center towards the plasma edge. This emphasises the importance of sheared $\vec{E} \times \vec{B}$ to be included into a theory. Most interesting is the comparison of the experimental and the theoretical radial eigen function profiles. This is shown in figure (4.8), in which the wave maximizes at $r = 3.3 \ mm$ and a quite well agreement is observed. The experimental data is obtained from Zhang, (1992).
Figure (4-7): The relationship between the radial eigen function $\varphi$ for $m = 1, 2, 3, 4$ and $5$ profiles predicted from a theoretical model and plasma beam radius $r(mm)$, (a): without $\vec{E} \times \vec{B}$ rotation; (b): with the shared $\vec{E} \times \vec{B}$ rotation and this curve is drawn using the theoretically normalised results, at a magnetic field $B = 875$ G.
Furthermore; Figure (4.7) displays the radial eigen function profiles obtained from the theory for modes \( m=1 \), to 5. The radial fluctuation profiles are normalized to an arbitrary value of 1, 0. As can be noticed, the mode moves further from the center of the plasma beam towards the edge when \( m \) increases. This indicates that the inclusion of \( \vec{E} \times \vec{B} \) into a theory has a significant influence in governing the drift wave evolution.

The predicted radial eigen function compared with the observed fluctuation level for \( m=1 \), and the magnetic field \( B = 875 \, G \). The parameter \( a = 7 \, mm \) is used in a typical case. These experimental data is obtained from Zhang (1992).

It is noticed that the wave amplitude is generally suppressed at the plasma beam edge in a magnetised plasma.

![Graph](image)

Figure (4.8): Comparison of experimental fluctuation level with theoretical eigen function calculated using \( a = 7 \, mm \) in a typical case for \( m=1 \).

Comparison of experimental fluctuation level with theoretical eigen function calculated using \( a = 7 \, mm \) in a typical case.
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5.1 Conclusions

Drift waves are excited evaluated in a plasma confinement systems. This is due to the presence of density, temperature profiles and pressure gradients in a laboratory condition which are inevitable, where the waves are supported by these gradients. In fact, The theoretical model is based on two-fluid equations, for either speed electrons or ions and simplified to the collisionless magnetised plasma regime. The inclusion of electron temperature gradient is found to be essential and must be taken into account in the theory. In addition, the $\mathbf{E} \times \mathbf{B}$ plasma rotation which arises when a radial electric field exist in an axial magnetic field is also included into a theoretical model. The effect of rotation is found to be Doppler shift the drift wave frequency so that plasma rotates as a solid body. The radial plasma density and temperature profiles which are required as input to the theoretical calculation are also presented and displayed in the present work. The theoretical model predicts the actual frequency, and the radial fluctuation profiles of the mode and hence, the position of the maximum wave amplitude can be determined. A comparison of experimental measurement with the theoretical prediction for radial variation of eigen function of density fluctuation is carried out. A good agreement is obtained between theory and experiment. The experimental data is obtained from an axial magnetised plasma of Zhang L. (1992) results. A matlab computer calculation is used for the differential equation for the radial distribution of the eigen function of density fluctuation. The relationship between the eigen function $\varphi$ obtained theoretically for $m = 1, 2, 3, 4$ and 5 profiles at magnetic field and plasma beam radius $r:mm$ are described in the present work. The effect of sheared $\mathbf{E} \times \mathbf{B}$ rotation on the drift wave, when the maximum value for the eigen function $\varphi$ for $m = 1$ with no $\mathbf{E} \times \mathbf{B}$ rotation is also found at $r = 2.9\ mm$. However, when sheared $\mathbf{E} \times \mathbf{B}$ is considered the maximum value is found at $r = 3.3\ mm$. That is the mode maximum moves a little further from the beam center towards the plasma edge. The result illustrates the importance of the effect of the electron-temperature and $\mathbf{E} \times \mathbf{B}$ rotation Doppler shift into a theoretical model.
5.2 Recommendations

The present work illustrated that the inclusion of electron temperature gradients and the $\vec{E} \times \vec{B}$ rotation are essential to the theoretical model. Thus, the present work suggests different parameters for future work should be included into a theory. This would be recommended as follows;

1- Measurement of variable pressure $\nabla p_i$ in the main equation and see the value of change that can occur with in the desired equation that results.

2- Measurement on a variable measuring primary frequency $v_i$ where you change the desired results and the value of the key equation to describe the plasma.

3- Measurement of a variable magnetic field $\nabla B$ to measure the possibility of the amendment to the plasma behavior in the impact of the change regarding the field.

4- Measurement of a variable electric field $\nabla E$ to measure the ability to modify behavior in the circle of the electric field effect on the particles.

5- Application general states equation entirely on completely spherical model to see the application mechanism and work on the logical explanation for the change happening in the plasma rotation.

6- The introduction of gravity variable in the equation full handle key that can change within a simple equation.

7- Application of the equation and change all belongings, considering that the electric and magnetic fields is regular.
The Reference List
The Reference List


